CMSC201
Computer Science I for Majors

Lecture 19 – Recursion

Last Class We Covered

- What makes "good code" good
  - Readability
  - Adaptability
  - Commenting guidelines
- Incremental development

#TBT

Function Returns

Any Questions from Last Time?

Today’s Objectives

- To introduce recursion
- To better understand the concept of “stacks”
- To begin to learn how to “think recursively”
  - To look at examples of recursive code
  - Summation, factorial, etc.

Introduction to Recursion

What is Recursion?

- In computer science, ***recursion*** is a way of thinking about and solving problems
- It's actually one of the central ideas of CS

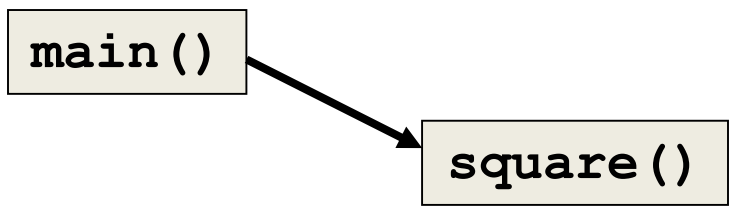
- In recursion, the solution depends on solutions to smaller instances of the same problem

Recursive Solutions

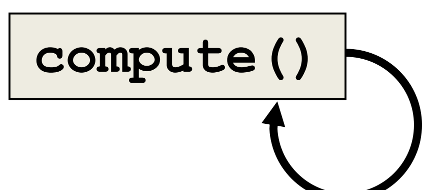
- When creating a recursive solution, there are a few things we want to keep in mind:
  1. We need to break the problem into smaller pieces of itself
  2. We need to define a "base case" to stop at
  3. The smaller problems we break down into need to eventually reach the base case

Normal vs Recursive Functions

- So far, we've had functions call other functions
  - For example, **`main()`** calls the **`square()`** function



- A recursive function, however, calls itself



Why Would We Use Recursion?

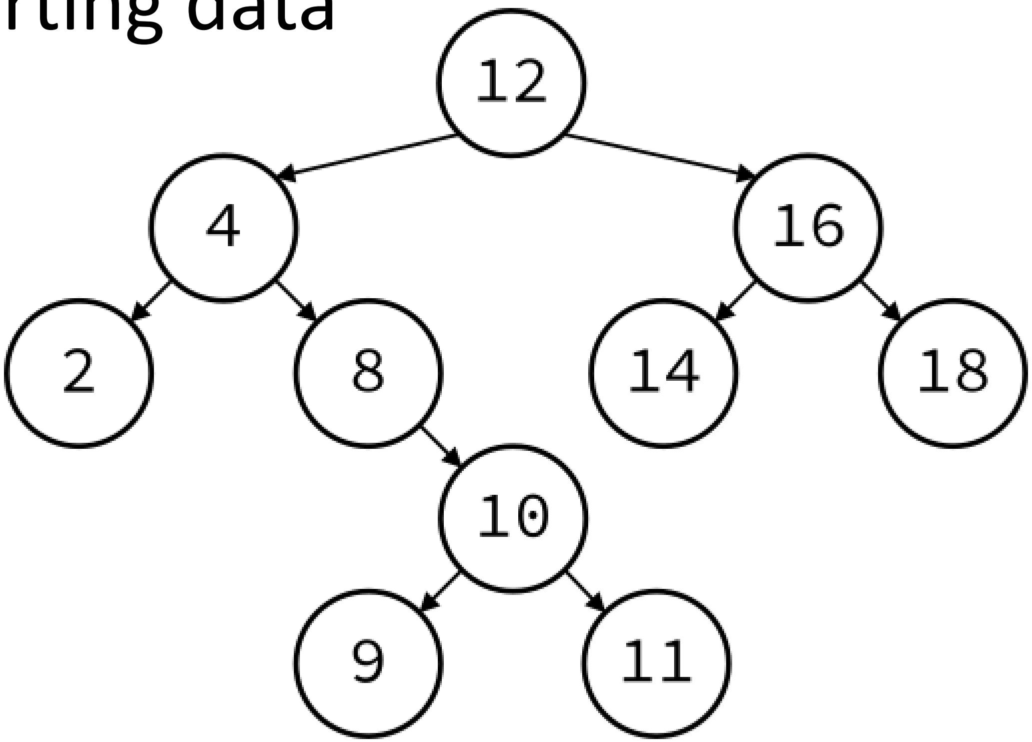
- In computer science, some problems are more easily solved by using recursive methods

- For example:
  - Traversing through a directory or file system
  - Traversing through a tree of search results
  - Some sorting algorithms recursively sort data

- For today, we will focus on the basic structure of using recursive methods

Recursion Examples

- Traversing a Binary Search Tree
- Sorting data



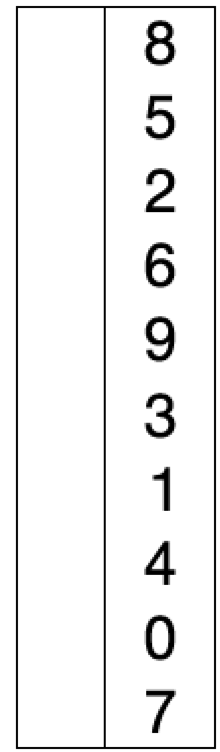


Image from wikimedia.org

Toy Example of Recursion

```
def compute(intInput):
    print(intInput)
    if (intInput > 2):
        compute(intInput-1)

def main():
    compute(50)

main()
```

What does this program do?

This program prints the numbers from 50 down to 2.

This is where the recursion occurs.

You can see that the **`compute()`** function calls itself.

Visualizing Recursion

- To understand how recursion works, it helps to visualize what's going on
- Python uses a ***stack*** to keep track of function calls
- A stack is an important computer science concept

Stacks

 Image from www.topwithcinnamon.com

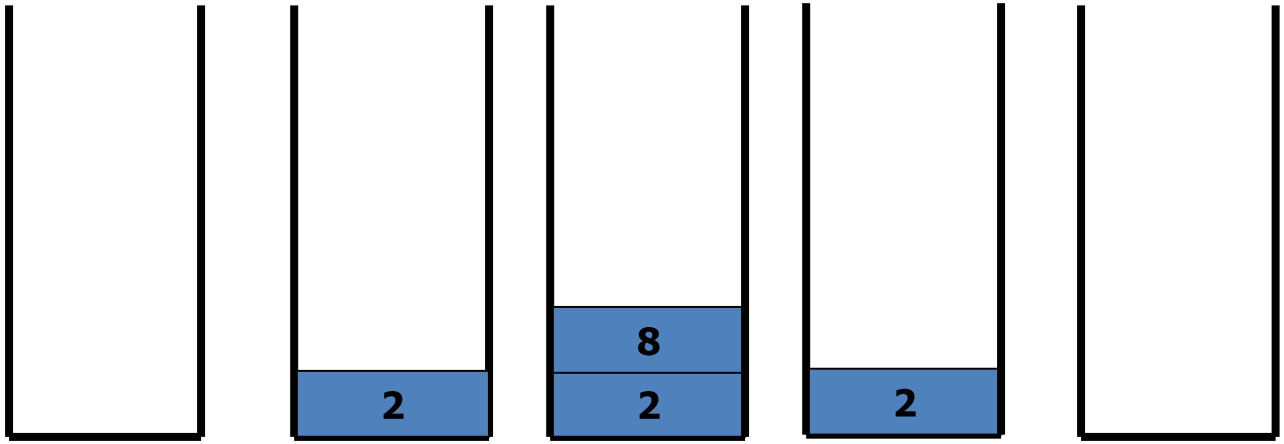
Stacks

- A stack is like a bunch of lunch trays in a cafeteria
- It has only two operations:
  - Push
    - You can push something onto the top of the stack
  - Pop
    - You can pop something off the top of the stack

- Let’s see an example stack in action

Stack Example

- The diagram below shows a stack over time
- We perform two pushes and two pops



Time: 0
Empty Stack

Time 1:
Push "2"

Time 2:
Push "8"

Time 3:
Pop

Time 4:
Pop

Stack Details

- In computer science, a stack is a *last in, first out* (LIFO) data structure

- It can store any type of data, but has only two operations: push and pop
- Push adds to the top of the stack, hiding anything else on the stack
- Pop removes the top element from the stack

Stack Details

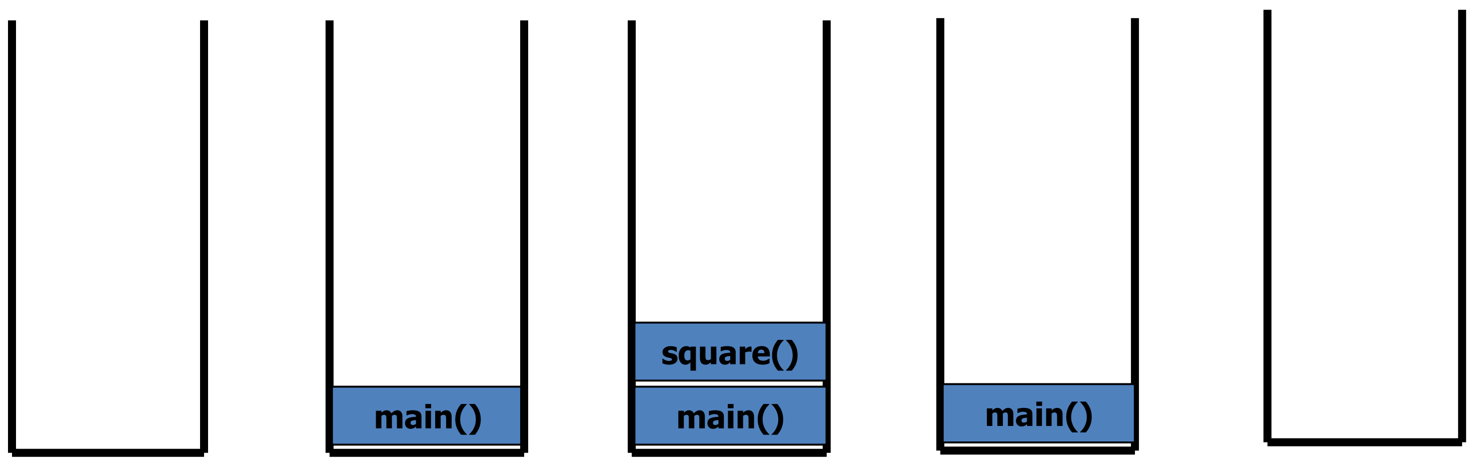
- The nature of the pop and push operations also means that stack elements have a natural order
- Elements are removed from the stack in the reverse order to the order of their addition
  - The lower elements are those that have been in the stack the longest

Stacks and Functions

- When you run your program, the computer creates a stack for you

- Each time you call a function, the function is pushed onto the top of the stack
- When the function returns or exits, the function is popped off the stack

Stacks and Functions Example



Time: 0
Empty Stack

Time 1:
Push: main()

Time 2:
Push: square()

Time 3:
Pop: square() returns a value. method exits.

Time 4:
Pop: main() returns a value. method exits.

Stacks and Recursion

- If a function calls itself recursively, you push another call to the function onto the stack
- We now have a simple way to visualize how recursion really works

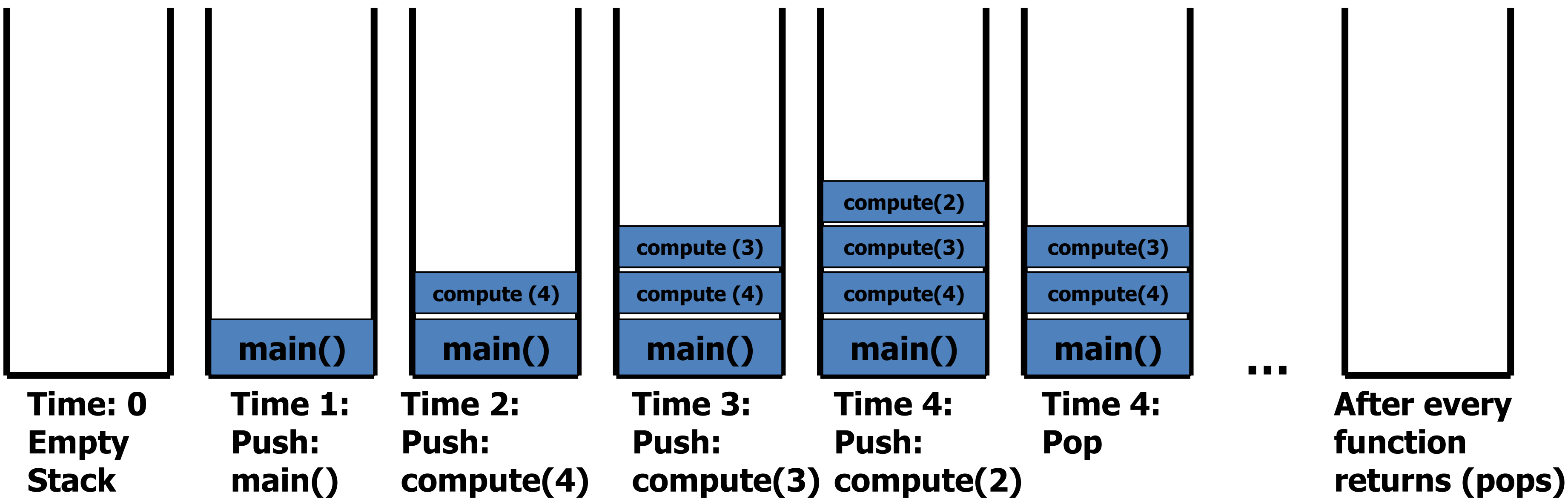
Toy Example of Recursion

```
def compute(intInput):
    print(intInput)
    if (intInput > 2):
        compute(intInput-1)

def main():
    compute(50)

main()
```

Here’s the code again.

Now, that we understand stacks, we can visualize the recursion.

Stack and Recursion in Action



Defining Recursion

"Cases" in Recursion

- A recursive function must have two things:

- At least one base case
  - When a result is returned (or the function ends)
  - "When to stop"
- At least one recursive case
  - When the function is called again with new inputs
  - "When to go (again)"

Terminology

```
def fxn(n):
    if n == 1:
        return 1
    else:
        return fxn(n - 1)
```

base case

recursive case

recursive call

- Notice that the recursive call is passing in simpler input, approaching the base case

Recursion Example

```
def sum(n):
    if n == 1:
        return 1
    else:
        return n + sum (n - 1)
```

- What is **`sum (1)`** ?
- What is **`sum (2)`** ?
- What is **`sum (100)`** ?
  - We at least know that it's **`100 + sum(99)`**

Recursion Example

```
def sum(n):
    if n == 1:
        return 1
    else:
        return n + sum (n - 1)
```

```
sum(3)
    3   + sum(2)
              2 + sum(1)
                      1
    3   +     2 +     1   =   6
```

Factorials

- $4! = 4 \times 3 \times 2 \times 1 = 24$

- Does anyone know the value of 9! ?
- 362,880

- Does anyone know the value of 10! ?
- How did you know?

Factorial

- $9! = 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$
- $10! = 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$
- $10! = 10 \times 9!$

- $n! = n \times (n - 1)!$

- That's a recursive definition!
  - The answer to a problem can be defined as a smaller piece of the original problem

Factorial

```
def fact(n):
    return n * fact(n - 1)

fact(3)
3 * fact(2)
3 * 2 * fact(1)
3 * 2 * 1 * fact(0)
3 * 2 * 1 * 0 * fact(-1)
...
```

What happened? What went wrong?

Factorial (Fixed)

```
def fact(n):
    if n == 0:
        return 1
    else:
        return n * fact(n - 1)

fact(3)
3 * fact(2)
3 * 2 * fact(1)
3 * 2 * 1 * fact(0)
3 * 2 * 1 * 1
```

Recursion Practice

Thinking Recursively

- Anything we can do with a **`while`** loop can also be accomplished through recursion
- Let's get some practice by transforming basic loops into a recursive function

- To keep in mind:
  - What is the base case? The recursive case?
  - Are we returning values, and if so, how?

Non-Recursive `getGrade()`

- Gets a grade between 0 and 100, inclusive

```
def getGrade():
    grade = int(input("Grade? "))
    while grade < 0 or grade > 100:
        print("That's not valid.")
        grade = int(input("Grade? "))
    return grade
```

- Transform this into a recursive function

Non-Recursive `sumList()`

- Sum the contents of a list together

```
def sumList(numList):
    total = 0
    for num in numList:
        total = total + num
    return total
```

- Transform this into a recursive function

Recursive Thinking

- Sometimes, creating a recursive function requires us to think about the problem differently
- What kind of base case do we need for summing a list together? How do we know we're "done"?
  - Instead of approaching the problem as before, think of it instead as adding the first element to the sum of the rest of the list

Recursive Summing

```
myList = [3,  5,  8,  7,  2,  6,  1]
           3 + [5, 8, 7, 2, 6, 1]
               5 + [8, 7, 2, 6, 1]
                   8 + [7, 2, 6, 1]
                          etc...
```

- What is the base case here?
- How does the recursive case work?

Announcements

- Project 2 out on Blackboard
  - Project due Friday, April 21st @ 8:59:59 PM
  - Uses 3D lists and file I/O

- Final exam is when?
  - Friday, May 19th from 6 to 8 PM

- Survey #2 out now, due Sunday @ 11:59 PM